LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – PHYSICS

SECOND SEMESTER – APRIL 2010

PH 2812 - MATHEMATICAL PHYSICS

Date & Time: 21/04/2010 / 1:00 - 4:00 Dept. No. Max. : 100 Marks

PART A

Answer ALL questions (10 x 2m =20m)

1) Integrate Re z from 0 to 1+2i along the shortest straight line path.
2) Determine the residue at Z= 0 and at Z = i of the complex function f (z) = $\frac{9Z+i}{Z(Z^2+1)}$.
3) Express the function f (t) = 2 if 0<t<π, f(t) = 0 if π<t<2π and f(t) = sin t if t >2π in terms of the unit step function.
4) Evaluate the Fourier transform of the first derivative of a function f (x).
5) Write down (i) one dimensional heat equation and (ii) two dimensional wave equation. Explain the symbols used.
6) What are the possible two initial conditions in the transverse vibration of a stretched string? Explain the symbols used.
7) Use the Rodrigue's formula for the Legendre polynomial to evaluate the 3rd order polynomial.
8) Write down the expression for the Bessel function $J_0(x)$ of zeroth order.
9) List the four properties that are required for a group multiplication.
10) What is irreducible representation of a group?

PART B

Answer any FOUR questions (4 x 7 ½ m = 30 m)

11) Show that the function u (x,y) = 4xy-3x+2 is harmonic. Construct the corresponding analytic function f (z) = u (x, y) + i v (x, y).
12) Solve the initial value problem $\frac{d^2y}{dt^2}+25y=10\cos 5t$, y (0) = 2, $\frac{dy(0)}{dt}=0$ by the Laplace transforms.
13) Using the method of separation of variables, solve the partial differential equations (i) $\frac{\partial u}{\partial x}=\frac{\partial u}{\partial y}$ and (ii) $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=(x+y)u$, where u is a function of x and y. (3+4 ½ m)
14) Establish the orthogonal properties of the Hermite polynomial, viz., $\int_{-\infty}^{\infty}\exp(-x^2)\,H_n(x)\,H_m(x)=\sqrt{\pi}\,2^n\,n!\,\delta_{mn}$ with the use of the generating function of the polynomial. (3+ 4 ½ m)

15) Work out the multiplication table of the symmetry group of the proper covering operations of a square (D_4). Write down all the subgroups and divide the group elements into classes. What are the allowed dimensionalities of the representation matrices of the group? (2+ 2+2+1 ½)

PART – C

Answer any FOUR questions (4 x 12 ½ m = 50 m)

16) (a) Using the contour integration, evaluate the following real integral

$$\int_0^{\infty} \frac{\cos 3x}{(x^2+1)(x^2+4)}dx$$ (6 ½ m)

(b) Evaluate the following contour integral:

$$\oint_C \frac{dz}{z^2+1}$$ with C: (a) |Z+i | = 1, and (b) |Z –i |= 1, counterclockwise. (6 m)

17) (a) Represent f (t) = sin 2t, $2\pi < t < 4\pi$ and f(t) = 0 otherwise, in terms of unit step function and find its Laplace transform. (6 ½ m)

(b) Solve y'' + 4 y' + 4 y = $t^2 e^{-2t}$ with initial conditions y (0) =0, y' (0) = 0 by Laplace transforms, where $y' = \frac{dy}{dt}$ and $y'' = \frac{d^2y}{dt^2}$. (6 m)

18) Solve the one- dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ by the separation of variable technique and the use of Fourier series. The boundary conditions are u(0,t) =0 and u(L,t) = 0 for all t and the initial conditions are u (x,0) = f (x) and $\frac{\partial u}{\partial t}$ = g(x) at t =0. (Assume that u (x,t) to represent the deflection of stretched string and the string is fixed at the ends x = 0 and x = L)

19) (a) Outline the the power series method of solving the Legendre differential equation

$$(1 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0 .$$ (6 ½ m)

(b) Establish the orthonormality relation $\int_{-1}^{+1} P_n(x)\, P_m(x)\, dx = \frac{2}{2n+1}\delta_{nm}$, where $P_n(x)$ is the Legendre polynomial of order n. (6 m)

20) (a) Obtain the transformation matrices of the symmetry elements (i) for the axis of symmetry and (ii) for the improper axis of symmetry . (6 ½ m)

(b)Enumerate and explain the symmetry elements of CO_2, H_2O and NH_3 molecules. (6 m)

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